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IMPROVED WIDE RANGE EXPRESSIONS FOR DISPLACEMENTS AND INVERSE DISPLACEMENTS FOR STANDARD FRACTURE TOUGHNESS SPECIMENS

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13. ABSTRACT *(Maximum 200 words)*
Wide range expressions (interpolating polynomials) for displacements for standard ASTM fracture testing specimens have been developed. The strategy for fitting was to fit displacements as a function of crack length for all of the specimens using a similar form of nondimensional displacement. Different forms appear in different ASTM Standards for the same or similar specimens. The criterion used to establish the degree of polynomial fit and number of significant figures for the coefficients of the polynomial was for the polynomial to agree with the best available numerical displacement solutions within one percent or better. Once the polynomials for displacement as a function of crack length were developed, inverted forms of the specific interpolating polynomials were obtained to determine relative crack length (a/W) as a function of displacement. Since the inverted polynomial will be used in tandem with the interpolating polynomial, the inverted polynomial was fit to the interpolating polynomial and not the numerical solution for displacement. Three forms of nondimensional displacements were used to invert the interpolating polynomials. The criterion used to establish the degree of polynomial and the number of significant figures of the coefficients of the polynomial was for the predicted relative crack length to agree with the actual relative crack depth to within 0.0005 W. A sensitivity analysis was performed to suggest which form of interpolating polynomial should be used for standards.

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INTRODUCTION

Wide range expressions (interpolating polynomials) for displacements for standard ASTM fracture testing specimens are used in the measurement of J_{Ic}, J-R, and R curve determination. They are used to predict crack extension using the compliance unloading methods. Although they have not been used in the measurement of K_{Ic} or for fatigue crack growth testing, such expressions may be useful in these test methods and could be incorporated in the standards for these tests in the future.

Attempts have been made in the past to develop appropriate expressions (refs 1-5). The work of Saxena and Hudak (ref 1) concentrated on the compact specimens and bending samples. The work of Kapp et al. (ref 2) attempted to address all of the specimens. The work of Wu (ref 3), Haagag and Underwood (ref 4), and Underwood et al. (ref 5), dealt with bending samples. The purpose of this report is to present expressions for all of the present standard specimens that are similar in form and accuracy.

PROCEDURE

The goals for the development of expressions are to develop expressions for displacement as a function of crack length that are accurate over the widest possible range of crack length and agree with the best available numerical analysis determination of displacement within about one percent or better. To develop the inverse expressions (where the crack length is calculated from displacement measurements), the following procedure was used. The developed displacement expression was used to determine the interpolated values of displacement at various crack lengths. This displacement was normalized to some form that can be used as a parameter to predict the crack length. Three forms of this parameter are presented for comparison: one suggested by Saxena and Hudak (ref 1) and two suggested by Kapp et al. (ref 2). Polynomials that are functions of the parameters were then fit to predict the relative crack length (a/W) to within 0.0005 W or better. Such accuracy is possible for all of the specimens considered at all of the displacement locations used for measurement with the exception of the arc tension sample. This is because the arc tension sample has many forms and has two additional geometric parameters (eccentric loading ratio (X/W) and radius ratio (r_1/r_2)). For this sample, it was not possible to predict relative crack length to better than about $\pm$ 0.003 W.

The nondimensional form for displacement that will enable the determination of accurate interpolating polynomials for opening displacement as a function of crack length is derived from the deep crack limit and short crack limits and was suggested in Reference 2. This form applies to crack-mouth-opening displacements for three-point bend samples.

$$\frac{E'Bv(1-a/W)^2}{15.8MW(a/W)} = f(a/W) \qquad (1)$$

where E' is the effective modulus (E for plane stress and $E/(1-\upsilon^2)$ for plane strain, (υ is Poisson's ratio)), B is the specimen's through thickness, v is the total displacement, a is the crack length, W is the specimen thickness, and M is the applied moment (PS/4 for three-point bending samples (where P is the applied load)). This form has finite limits as the relative crack length (a/W) approaches both the short crack limit (a/W = 0) and the deep crack limit (a/W = 1).

For load-line displacements for the compact and disk compact samples and crack-mouth-opening displacements for arc tension samples, the form given by Eq. (1) is used, with M replaced by PW for both compact samples and with P(X+W/2+a/2) for arc tension samples. A second modification for these samples is that the a/W term in the denominator is removed. This term comes from the short crack limit which is unknown for the compact samples and is too complicated for the arc tension sample. For displacements other than load-line for compact specimens, the W term in the denominator is replaced by

$(1+\Delta)W$, where Δ is the distance from the load line to the point where the displacement is being measured. Finally, for load-line displacements for three-point bending samples, the denominator includes an M^2 term. The explicit form for each specimen and displacement location is presented below.

Another form of nondimensional displacement has been suggested as (ref 6)

$$\frac{E'Bv(1-a/W)^2}{15.8MW(1+a/W)^2(a/W)} = f(a/W) \tag{a}$$

which is merely Eq. (1) with an additional $(1+a/W)^2$ term in the denominator. This term was added in Reference 6 for expediency in fitting and is not based on any derived limit solution. Since the goal of this report was to determine interpolating polynomials for displacements that are accurate to within about one percent or better, the $(1+a/W)^2$ term was not needed. The form of Eq. (1) was used for all of the interpolating polynomials.

For the inverse expressions, three parameters have been suggested. The familiar form from Saxena and Hudak is an empirical parameter

$$U = \frac{1}{\left(\frac{EBv}{P}\right)^{1/2} + 1} \tag{2}$$

and $a/W = f(U)$. This parameter has been used for many samples. It has finite limits as the value of displacement approaches zero, a/W approaches zero and U approaches one. When v approaches infinity, a/W approaches one and U approaches zero. Although the parameter has finite limits, as the displacement approaches its two extremes, this parameter is unlike the nondimensional form of displacement of Eq. (1) in that it was not derived from some known limiting solution. A derived form of an inverse parameter was presented in Reference 2 based on the deep crack limit

$$U = 1 - \left(\frac{15.8M}{E'BWv}\right)^{1/2} \tag{3}$$

This form of U in Eq. (3) is the deep crack limit. As v approaches infinity, U approaches a/W and is found by algebraic manipulation of Eq. (1) without the a/W term in the denominator. Although this form was first presented in Reference 2, it was dismissed as being an inadequate parameter, since it has an infinite limit as the displacement approaches zero. The form chosen in Reference 2 as an appropriate form was a combination of the two forms presented above

$$U = \frac{1}{\left(\frac{15.8M}{E'BWv}\right)^{1/2} + 1} \tag{4}$$

This parameter has finite limits at both extremes. As v approaches infinity, a/W approaches one and U approaches one, and as v approaches zero, a/W approaches zero and U approaches zero. Using this form, the coefficients of the interpolating polynomial are much simpler than the coefficients obtained when using the form in Eq. (2).

In this report, all three forms are used. The restrictions on the use of the parameter in Eq. (3) (the parameter blows up when the displacement and a/W approach zero) are meaningless since the forms of Eq. (2) or (4) make their use at values of v approaching zero suspect. To demonstrate, the crack-mouth-opening displacements for the SE(B) sample from Reference 7 are used to develop values of the U parameters of Eqs. (2) and (3). These parameters are plotted against a/W in Figure 1. Also shown in the figure are the third, fourth, and fifth order least squares interpolating polynomials for each parameter. It is clear that for both of these parameters, the short crack limit is reached. However, the manner in which the limit is approached is inappropriate. All interpolating polynomials for both parameters predict a negative crack length when the value of the parameter approaches its short crack limit. This means that none of these parameters are appropriate for predicting the crack length when the crack is small. Therefore, it was decided to limit the applicability of either of these two parameters to predicting the minimum crack length for which there is accurate numerical determination of the value of the displacement.

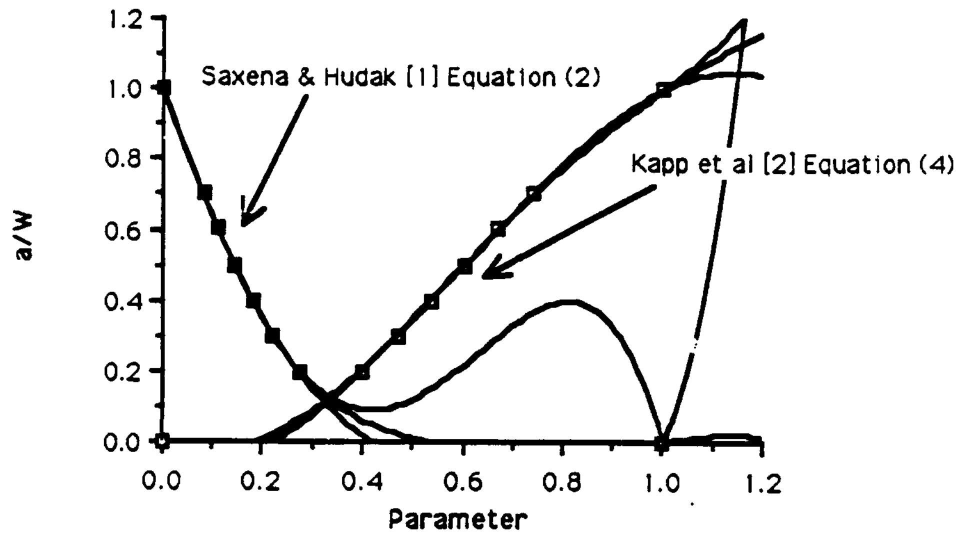

Figure 1. Comparison of the U parameters for SE(B) crack-mouth-opening displacements for $0 \leq a/W \leq 1.0$.

Since the two parameters of Eqs. (3) and (4) were not used at very short crack lengths, the derived parameter given as Eq. (3) can also be used, since this parameter only blows up when the displacement is very small or when the crack length approaches zero. All three parameters were evaluated for the SE(B) crack-mouth-opening displacements for values of a/W between 0.2 and 1.0 and are plotted in Figure 2. Also plotted in the figure is the fifth order least squares interpolating polynomials for each parameter. This plot shows that the interpolating polynomials are appropriate in this case.

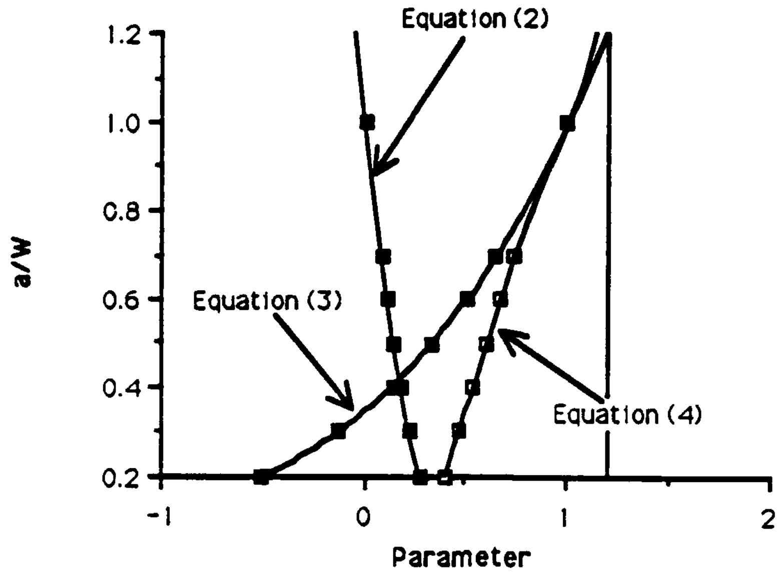

Figure 2. Comparison of the U parameters for SE(B) crack-mouth-opening displacements for $0.2 \leq a/W \leq 1.0$.

RESULTS AND DISCUSSION

Displacements as a Function of Crack Length

The appropriate form of displacements as a function of crack length is given in Eq. (1). The manner in which the left-hand side of Eq. (1) (f(a/W)) was determined was to normalize the numerical data from the best available source according to the left-hand side of Eq. (1). The discreet values of f(a/W) determined in this manner were coupled with the limit solution and the intermediate values of f(a/W) were determined by least squares fitting. All of the results can be summarized as follows:

$$f(a/W) = A_0 + A_1(a/W) + A_2(a/W)^2 + A_3(a/W)^3 + A_4(a/W)^4 + A_5(a/W)^5 \qquad (5)$$

The explicit form of f(a/W) for all ot the specimens and the values of the coefficients are given in Table 1.

Some discussion of these results is necessary. First, not all of these polynomials were developed for this report. In the table, if a reference number appears next to the specimen and location designation, then the polynomial is from that reference. For the SE(B) (CM) displacements, a totally different form of interpolating polynomial appears in E-813 and E-1152. This is the interpolating polynomial from Reference 8. This polynomial is wrong. It does not approach the short crack limit correctly and was fit to the uncorrected displacement solution in Reference 7. The corrected displacements for the SE(B) (CM) were published in the errata of Reference 7. Furthermore, the interpolating polynomial reported in the table for the SE(B) (CM) was not determined by fitting to all of the numerically determined displacements in Reference 7. The displacement at a/W = 0.8 in the errata to Reference 7 is of questionable accuracy. All of the other numerically determined displacements fall smoothly on a curve that approaches both the deep and short crack limits, while the numerically determined value of displacement at a/W = 0.8 clearly falls off of the smooth curve. It was decided to develop the polynomial reported in the table by excluding the numerical displacement solution at a/W = 0.8.

The accuracy column addresses the accuracy of the interpolating polynomial with respect to the numerically generated displacements. No attempt is made to give a number to the absolute accuracy of the polynomials (the accuracy between the interpolating polynomials and the true displacement solutions). The solutions used for the fitting are as follows: SE(B) (CM) (ref 7); SE(B) (LL) (ref 4); C(T), (LL), and (CM) and D(T), (LL) and (CM) (ref 9); C(T) and C(W) (LL + 0.1576 W) (ref 10); and A(T) (refs 2 and 11). It is clear that for all of the interpolating polynomials, the accuracy requirement of within one percent is met with the possible exception of the A(T) samples. As previously stated, the A(T) sample includes two additional geometric variables that must also be interpolated. The interpolation in terms of eccentric loading (X/W) is not performed. There are only two standard specimens (X/W = 0 and X/W = 0.5). To maximize accuracy, a separate polynomial for each X/W is included. This eliminated one of the interpolating variables. The other geometric variable of radius ratio (r_1/r_2) had to be addressed empirically. The simplest way of accounting for the r_1/r_2 effect is with the use of the addition of the linear function r_1/r_2 as described in the table. This is just approximate, since the r_1/r_2 effect is a function of a/W. At short crack lengths, the r_1/r_2 dependence is very strong, yet at deep crack lengths, the r_1/r_2 effect is zero. In between these limits, the effect of r_1/r_2 is nearly constant and linear, allowing the use of such a simple correction. However, there is a price that is to be paid, in that with the linear correction reported in the table, the nondimensional form of f(a/W) does not approach the deep crack limit for a specimen with r_1/r_2 other than 1.0. This means that the interpolating polynomials for the A(T) samples are only accurate over the range of a/W from 0.2 to 0.8.

Crack Length as a Function of Displacement

The three forms for inverting polynomials given in Table 1 have been used. These are the parameters given as Eqs. (2), (3), and (4). As with the polynomials in Table 1, the crack length is estimated by a polynomial

$$a/W = A_0 + A_1U + A_2U^2 + A_3U^3 + A_4U^4 + A_5U^5 \tag{6}$$

where U is one of the parameters given in Eq. (2), (3), or (4). The major difference here is that the parameter U is not based on the numerical results themselves, but by the calculated values of displacement given by the polynomials in Table 1. This is done because of the manner in which it is anticipated that the expressions will be used. First, at a known value of a/W, the displacement of a specimen will be measured. This will enable the calculation of the effect modulus E′ by using one of the polynomials in Table 1. The polynomials of the form of Eq. (6) will then be used to calculate crack length. Since the specimen will be calibrated by the use of the interpolating polynomials, errors in crack length measurement will be minimized if the inverse expressions (Eq. (6)) are fit to the calculated values of displacement rather than the numerically determined values of displacement. The values of the coefficients in Eq. (6) for each of the three forms of U given above are in Tables 2, 3, and 4.

A general discussion of the polynomials are that the required accuracy of ± 0.0005 W is obtained using all forms of the U parameter with the exception of the SE(B), load-line displacements, and the A(T) samples. In the case of SE(B) load-line displacements, the total displacement is the displacement due to the presence of the crack plus the deflection with no crack present. All of the U parameters assume that the crack length is a function of the square root of displacement. For the SE(B) load-line, the crack length should be a function of the displacement plus something, all under the square root symbol (ref 2). This means that a parameter other than the U parameters of Eq. (2), (3), or (4) should be used. However, it was possible to predict crack lengths within 0.0005 W with a fifth order polynomial and U according to Eq. (3). Using either Eq. (2) or (4) for the U parameter, only a fourth order polynomial is reported in the tables. This is because the use of a fifth order polynomial with the Eq. (2) or (4) form of U did not increase the accuracy, and a plot of these polynomials shows that the limits are approached in an inappropriate manner. For the A(T) specimens, the errors are due to the fact that the r_1/r_2 effect on the U parameters is a function of crack length. Since we are attempting to calculate crack length, we cannot use it to account for the errors in U parameter and then use the U parameter to calculate crack length. Therefore, we must account for the r_1/r_2 effect in an average way. This is the reason for the inferior accuracy of the A(T) sample expressions.

Comparing the use of the different forms of U to predict crack length, the following general observations can be made. Using the form of U in Eq. (2) results in inverse polynomials with coefficients that are roughly geometrically increasing. A_0 is always about one, A_1 is about five, A_2 is about ten, A_3 is in the hundreds, A_4 is in the thousands, etc. This means that all of the terms in the polynomials using this U parameter contribute about the same to the calculation of a/W. Accordingly, this indicates that one must be careful when calculating crack length using such polynomials. Using the U parameter of Eq. (4), and in particular using Eq. (3), all of the coefficients are of approximately the same order. In these cases, the terms increasing in order involving U contribute less to the total calculation. For these cases, the first few terms are more important than the latter terms. With the use of the Eq. (3) or (4) forms of U, most of the calculated a/W is obtained with the first few terms, suggesting that these forms may be more forgiving of calculation error.

The calculation error discussion is presented because to calculate a/W using the inverted polynomials in Table 2, 3, or 4, we do not use a directly measured independent variable to calculate the dependent variable. For example, to use the interpolating polynomials in Table 1, we measure the crack length, divide by a constant, and use that as the independent variable. When using the inverse polynomials, we measure a slope (v/P or P/v), multiply by some constants, invert the result, take a square root, add or subtract a constant, and invert the result just to get the independent variable. Many calculations are made before we use the interpolating polynomial to obtain the desired result--in this case, the crack length. The possibility for calculation error in the evaluation of the independent variable (the U parameter) is great. This leads me to argue that the best form of the parameter U to use for calculating a/W is that form of U most resistant to errors in the parameter U itself, since all of the U parameters can be used to calculate a/W accurately, if everything is done properly.

It is beyond the scope of this work to perform a detailed analysis of calculation error on all of the interpolating polynomials in Tables 2, 3, and 4. An indication of the results of such an error analysis can be observed with a few numerical experiments on the use of all three forms of U for a single specimen. The SE(B) crack-mouth-opening displacement expressions are chosen arbitrarily.

It is assumed that several types of calculation errors can occur. First, it is observed that the coefficients of the interpolating polynomial necessary when using the U parameter in the form of Eq. (2) have as many as six significant figures. The other forms require fewer. In particular, when using the Eq. (3) form of U, there are only three significant figures except for the constant term which has four. It should be pointed out that any constant term in any interpolating polynomial that has four decimal places, the fourth decimal place was added artificially to minimize errors. One may be tempted to see how accurate the expressions are when all of the coefficients are rounded to three significant figures. Table 5 shows the comparison of calculating a/W with all forms of U and rounded coefficients on the interpolating polynomials. The other forms of calculation error are rounding of the calculated U parameter. In Table 6, the effects of rounding the U parameter to six, four, three, and two decimal places are presented. The effects of a relative percentage error in calculating the various U parameters are presented in Table 7. Finally, the effect of an absolute error of 0.0001, 0.001, or 0.01 in the calculation of U are presented in Table 8.

Discussing the effects of rounding the coefficients first, Table 5 presents two sets of calculations. The first data set uses the correct form of U rounded to four decimal places (four decimal places were used in the least squares fitting), and calculates a/W using interpolating polynomials with all of the significant figures from Table 2, 3, or 4. The results show that a/W is predicted to within 0.0002 W for the

Eq. (3) or (4) forms of U and within 0.0001 W for the Eq. (2) form of U. The second set of data again uses the correctly calculated U to four decimal places, but now with the coefficients of the interpolating polynomials rounded to three significant places. The errors in calculating a/W are, of course, increased. In this case, the maximum error is 0.0004 W for the Eq. (3) form, -0.0007 W for the Eq. (2) form, and -0.0023 W for the Eq. (4) form. This means that the Eq. (3) form of U is most insensitive to the effects of significant figures of the interpolating polynomials.

The effects of rounding the calculated value of U using all of the significant figures in the coefficients of the interpolating polynomials are presented in Table 6. There are three sets of calculations: one for each form of U. For each case, the calculated U is rounded to six, four, three, or two decimal places. The first set of calculations deals with the form of U given by Eq. (2). The effects of rounding the U parameter are interesting. First, when rounded to four decimal places, the error calculated in a/W is ± 0.0001 W, but when rounded to six decimal places, the error increases to ± 0.0002 W. When rounded to three or two decimal places, the errors are ± 0.0008 and 0.009 W (maximum), respectively. Using the form of U given by Eq. (4) (the second set of data), the same accuracy ± 0.0002 W is obtained with rounding to either six or four decimal places. The error is increased to -0.0008 W maximum when U is rounded to three decimal places, and when rounded to two decimal places, the maximum error is -0.0062 W. When using the form of U given by Eq. (3), the error is ± 0.0003 W when the U parameter is rounded to either six, four, or three decimal places and is only -0.0028 W maximum when the parameter is rounded to only two decimal places. These results show clearly that the form of U parameter that is least sensitive to rounding of the U parameter is the form given in Eq. (3).

The next form of calculation error addressed is calculating the U parameter correctly to six decimal places, adding either 0.1, 1, or 5 percent, then rounding to four decimal places. The errors in the calculation of a/W for each form of U are presented in Table 7. The first set of data uses U in the form of Eq. (2). Miscalculating the U parameter by 0.1 percent will introduce a maximum error of -0.0006 W in the calculation of a/W, a 1 percent miscalculation results in an error of about -0.004 W at all crack lengths, and a 5 percent miscalculation results in about a -0.02 W error at all crack lengths. With the form of U parameter of Eq. (4), a 0.1 percent miscalculation of U causes an error that varies from 0.0003 to 0.001 W, a 1 percent miscalculation results in an error that varies from 0.005 to 0.0098 W, and a 5 percent miscalculation results in an error that varies from 0.027 to 0.048 W. Considering the form of U given by Eq. (3), a relative miscalculation of 0.1 percent causes errors that vary from 0 to 0.0006 W, a 1 percent miscalculation causes errors that vary from -0.001 to 0.0049 W, and a 5 percent miscalculation causes errors from -0.005 to 0.024 W. It is difficult to say which form of U is least sensitive to relative miscalculations of the U parameter. It can be argued that when using U in the form of Eq. (2), the errors are more consistent (they are roughly equal at all crack lengths). For consistency to be a concern, however, it must be known that a miscalculation has been made. The meter that must be used to determine which form is more resistant to relative miscalculation errors is to determine which form will give the most accurate estimate of a/W over the widest range of a/W. Using this, it is clear that the form of U given by Eq. (3) is the most accurate.

The final kind of calculation error that may be encountered is an absolute error. Table 8 examines this. The absolute calculation errors are introduced in the following manner. First, the correct form of the parameter is determined, then added to it are either 0.0001, 0.001, or 0.01. The result is rounded to four decimal places, and the erroneous value of U is used to calculated a/W. Comparisons of the effects of these kinds of errors are presented in the same manner as above. Using the form of U given by Eq. (2), a miscalculation error of 0.0001 causes errors in the calculation of a/W of as much as -0.0005 W, a miscalculation of 0.001 in U causes errors that vary from -0.0018 to -0.0036 W, and a miscalculation of 0.01 in U results in errors of between -0.0168 and -0.0344 W. Using the form of U given by Eq. (4), a miscalculation of 0.0001 increases the accuracy of the interpolating polynomial to ± 0.0001 W. A miscalculation of 0.001 in U causes a uniform error of about 0.0013 W, and a miscalculation of 0.01 in U results in an error of about 0.014 W. With the form of U in Eq. (3), the effect of a miscalculation of U by

0.0001 has no effect on the accuracy of the predicted a/W. A miscalculation of 0.001 in the form of U of Eq. (3) causes errors that vary from 0.0003 to 0.0008 W, and a miscalculation of 0.01 results in errors ranging from 0.0024 to 0.0075 W. Clearly, the form that is most resistant to absolute miscalculation errors is the form given by Eq. (3).

SUMMARY AND CONCLUSIONS

Wide range expressions have been developed for all of the standard specimens presently used to determine K_{Ic}, J_{Ic}, K-R, and J-R. One set of expressions give displacements as a function of crack length. A second, third, and fourth group give crack length as a function of displacement using three ways of normalizing the displacements. All of the expressions for displacements as a function of crack length agree with numerical displacement solutions to within about one percent or better. Also, all of the expressions that give crack length as a function of displacements will give calculated values of a/W within 0.0005 W or better. The interpolating polynomials for displacements as a function of crack length and crack length as a function of displacement are used in tandem. That is, the inverse expressions for crack length as a function of displacements were fit such that they invert the interpolating polynomials for displacement as a function of crack length. Since all three forms of the inverse expressions give results of about the same accuracy, an analysis was performed to determine which form of normalized displacement should be used to calculate crack length. The form that appears to give the best results is derived from the deep crack limit solution.

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Table 1. Interpolating Polynomials for Displacement = f(a/W).

Specimen	f(a/W)	A_0	A_1	A_2	A_3	A_4	A_5
SE(B) (CM)	$\frac{E'Bv(1-a/W)^2}{3.95P(S/W)(a/W)}$	2.212	-7.232	22.73	-38.92	32.95 ±0.8% for any a/W	-10.7
SE(B) (LL) [4]	$\frac{E'Bv(1-a/W)^2}{P(S/W)^2}$	1.193	-1.980	4.478	-4.443	1.739 ±1.1% for any a/W	0.00
C(T) (CM)	$\frac{E'Bv(1-a/W)^2}{19.75P}$	0.5	0.192	1.385	-2.919	1.842 ±0.1% for 0.2≤ a/W≤1.0	0.0
C(T) (LL) [2]	$\frac{E'Bv(1-a/W)^2}{15.8P}$	0.121	1.21	-0.159	-1.47	1.30 ±0.2% for 0.2≤ a/W≤1.0	0.0
C(T) (LL+0.1576W)	$\frac{E'Bv(1-a/W)^2}{18.29P}$	0.364	0.678	0.517	-2.152	1.593 ±0.2%, 0.35≤ a/W≤1.0	0.0
C(W) (LL+0.1576W)	$\frac{E'Bv(1-a/W)^2}{18.29P}$	0.094	1.704	-1.255	-0.574	1.03 ±0.2%, 0.35≤ a/W≤1.0	0.0
D(T) (CM)	$\frac{E'Bv(1-a/W)^2}{19.75P}$	0.469	-0.056	1.86	-2.06	0.789 ±0.2% for 0.2≤ a/W≤1.0	0.0
D(T) (LL) [2]	$\frac{E'Bv(1-a/W)^2}{15.8P}$	0.104	1.11	-0.262	0.0247	0.0223 ±0.3% for 0.2≤ a/W≤1.0	0.0

Table 1. (Continued)

Specimen	f(a/W)	A_0	A_1	A_2	A_3	A_4	A_5
A(T) (CM,X/W=0)	$\left(\frac{E'Bv(1-a/W)^2}{P(2X/W+1+a/W)} - 0.43(1-r_1/r_2)\right)$						
		0.542	13.137	-12.316	6.576	0.0 ±1.4% for $0.2 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$	0.0
A(T) (CM,X/W=0.5)	$\left(\frac{E'Bv(1-a/W)^2}{P(2X/W+1+a/W)} - 0.45(1-r_1/r_2)\right)$						
		0.399	12.63	-9.838	4.66	0.0 ±1.6% for $0.2 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$	0.0

LEGEND: (CM)= Crack Mouth; (LL) = Load Line; (LL+.1576W) = Load Line + 0.1576W (V_1 location in E561)

Table 2. Interpolating Polynomials for a/W = f(Displacement).

$$U = \frac{1}{\left(\frac{EBv}{P}\right)^{\frac{1}{2}} + 1}$$

Specimen	A_0	A_1	A_2	A_3	A_4	A_5	
SE(B) (CM)	1.000	-3.937	3.361	-11.413	87.634	-150.111	±0.0001W a/W ≥ 0.2
SE(B) (LL)	1.193	-1.980	4.478	-4.443	1.739	0.0	±0.0013W a/W ≥ 0.2
C(T) (CM)	1.000	-4.500	13.157	-172.551	879.944	-1514.671	±0.0002W a/W ≥ 0.2
C(T) (LL)	1.000	-3.999	9.727	-92.716	416.559	-591.161	±0.0001W a/W ≥ 0.2
C(T) (LL+0.1576W)	1.000	-5.005	31.752	-353.6	1664.381	-2747.965	±0.0003W a/W ≥ 0.35
C(W) (LL+0.1576W)	1.000	-5.007	34.726	-411.71	2121.025	-3815.898	±0.0001W a/W ≥ 0.35
D(T) (CM)	1.000	-4.459	2.066	-13.041	167.627	-481.4	±0.0001W a/W ≥ 0.2
D(T) (LL)	1.000	-3.98	2.338	-0.155	42.656	-91.274	±0.0001W a/W ≥ 0.2
A(T) (CM,X/W=0) $\left(U = \frac{1}{\left(\frac{EBv}{P(1+0.101(1-r_1/r_2))}\right)^{\frac{1}{2}} + 1}\right)$							
	0.989	-3.463	-0.171	24.354	-72.805	84.375	±0.003W, 0.2≤ a/W≤0.8 and $r_1/r_2 \geq 0.4$

Table 2. (Continued)

Specimen	A_0	A_1	A_2	A_3	A_4	A_5	
A(T) (CM,X/W=0.5)	$\left(U = \frac{1}{\left(\frac{EBv}{P(1+0.108(1-r_1/r_2))}\right)^{\frac{1}{2}} + 1}\right)$						
	0.986	-4.082	-5.065	86.819	-313.338	429.101	$\pm 0.003W$, $0.2 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$

LEGEND: (CM)= Crack Mouth; (LL) = Load Line; (LL+.1576W) = Load Line + 0.1576W (V_1 location in E561)

Table 3. Interpolating Polynomials for a/W = f(Displacement).

Specimen	Parameter	A_0	A_1	A_2	A_3	A_4	A_5
SE(B) (CM)	$U = \dfrac{1}{\left(\dfrac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}} + 1}$	-0.2338	1.356	-3.443	11.148	-12.38	4.553
						±0.0002W, a/W ≥ 0.2	
SE(B) (LL)	$U = \dfrac{1}{\left(\dfrac{0.9875(S/W)^2P}{EBv}\right)^{\frac{1}{2}} + 1}$	-4.7656	16.508	-17.046	6.3041	0.0	0.0
						±0.005W, a/W ≥ 0.2	
C(T) (CM)	$U = \dfrac{1}{\left(\dfrac{19.75P}{EBv}\right)^{\frac{1}{2}} + 1}$	-1.273	3.713	-2.274	-3.898	-5.561	2.497
						±0.0002W, a/W ≥ 0.2	
C(T) (LL)	$U = \dfrac{1}{\left(\dfrac{15.8P}{EBv}\right)^{\frac{1}{2}} + 1}$	0.9487	-8.164	24.709	-28.316	14.35	-2.528
						±0.0002W, a/W ≥ 0.35	
C(T) (LL+0.1576W)	$U = \dfrac{1}{\left(\dfrac{18.29P}{EBv}\right)^{\frac{1}{2}} + 1}$	0.428	-6.119	20.187	-20.968	7.472	0.0
						±0.0001W, a/W ≥ 0.35	
C(W) (LL+0.1576W)	$U = \dfrac{1}{\left(\dfrac{18.29P}{EBv}\right)^{\frac{1}{2}} + 1}$	3.123	-22.947	63.258	-76.964	44.169	-9.639
						±0.0001W, a/W ≥ 0.35	

Table 3. (Continued)

Specimen	Parameter	A_0	A_1	A_2	A_3	A_4	A_5
D(T) (CM)	$U = \frac{1}{\left(\frac{19.75P}{EBv}\right)^{\frac{1}{2}} + 1}$	-3.7073	21.619	-51.665	68.65	-46.339	12.442
							±0.0002W, $a/W \geq 0.2$
D(T) (LL)	$U = \frac{1}{\left(\frac{15.8P}{EBv}\right)^{\frac{1}{2}} + 1}$	0.1122	-2.137	9.072	-10.04	4.839	-0.846
							±0.0003W, $a/W \geq 0.35$
A(T) (CM,X/W=0)	$\left(U = \frac{1}{\left(\frac{7.9P(2X/W+1)(1+0.101(1-r_1/r_2))}{EBv}\right)^{\frac{1}{2}} + 1}\right)$	0.519	-4.282	12.715	-13.279	6.573	-1:248 ±0.003W, $0.2 \leq a/W \leq 0.8$ ±0.0015W, $0.3 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$
A(T) (CM,X/W=0.5)	$\left(U = \frac{1}{\left(\frac{7.9P(2X/W+1)(1+0.108(1-r_1/r_2))}{EBv}\right)^{\frac{1}{2}} + 1}\right)$	1.013	-8.528	27.032	-36.194	24.358	-6.702 ±0.0036W, $0.2 \leq a/W \leq 0.8$ ±0.0015W, $0.3 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$

LEGEND: (CM)= Crack Mouth; (LL) = Load Line; (LL+.1576W) = Load Line + 0.1576W (V_1 location in E561)

Table 4. Interpolating Polynomials for a/W = f(Displacement).

Specimen	Parameter	A_0	A_1	A_2	A_3	A_4	A_5
SE(B) (CM)	$U = 1 - \left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}$	0.3458	0.377	0.218	0.076	0.017	0.0
						$\pm 0.0002W$, $a/W \geq 0.2$	
SE(B) (LL)	$U = 1 - \left(\frac{0.9875(S/W)^2P}{EBv}\right)^{\frac{1}{2}}$	-0.1135	2.313	-4.222	6.328	-4.687	1.381
						$\pm 0.0005W$, $a/W \geq 0.2$	
C(T) (CM)	$U = 1 - \left(\frac{19.75P}{EBv}\right)^{\frac{1}{2}}$	0.2331	0.593	0.103	0.458	-0.745	0.358
						$\pm 0.0002W$, $a/W \geq 0.2$	
C(T) (LL)	$U = 1 - \left(\frac{15.8P}{EBv}\right)^{\frac{1}{2}}$	0.3222	0.417	0.280	0.078	-0.245	0.148
						$\pm 0.0003W$, $a/W \geq 0.35$	
C(T) (LL+0.1576W)	$U = 1 - \left(\frac{18.29P}{EBv}\right)^{\frac{1}{2}}$	0.258	0.571	-0.057	0.941	-1.300	0.588
						$\pm 0.0002W, 0.6 \geq a/W \geq 0.35$ $\pm 0.001W$, $a/W \geq 0.35$	
C(W) (LL+0.1576W)	$U = 1 - \left(\frac{18.29P}{EBv}\right)^{\frac{1}{2}}$	0.302	0.423	0.282	0.217	-0.429	0.205
						$\pm 0.0003W$, $a/W \geq 0.35$	

Table 4. (Continued)

Specimen	Parameter	A_0	A_1	A_2	A_3	A_4	A_5
D(T) (CM)	$U = 1 - \left(\frac{19.75P}{EBv}\right)^{\frac{1}{2}}$	0.2598	0.540	0.0829	0.176	-0.0535	-0.00504 $\pm 0.0002W$, $a/W \geq 0.2$
D(T) (LL)	$U = 1 - \left(\frac{15.8P}{EBv}\right)^{\frac{1}{2}}$	0.333	0.389	0.212	0.078	0.012	0.0 $\pm 0.0002W$, $a/W \geq 0.35$
A(T) (CM,X/W=0)	$U = 1 - \left(\frac{7.9P(2X/W+1)(1+0.101(1-r_1/r_2))}{EBv}\right)^{\frac{1}{2}}$	0.267	0.355	0.248	-0.084	0.395	-0.194 $\pm 0.002W$, $0.2 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$
A(T) (CM,X/W=0.5)	$U = 1 - \left(\frac{7.9P(2X/W+1)(1+0.108(1-r_1/r_2))}{EBv}\right)^{\frac{1}{2}}$	0.295	0.373	0.207	-0.011	0.331	-0.211 $\pm 0.0036W$, $0.2 \leq a/W \leq 0.8$ $\pm 0.0015W$, $0.3 \leq a/W \leq 0.8$ and $r_1/r_2 \geq 0.4$

LEGEND: (CM)= Crack Mouth; (LL) = Load Line; (LL+.1576W) = Load Line + 0.1576W (V_1 location in E561)

Table 5. Effects of Rounding the Coefficients of Interpolating Polynomials for SE(B) Crack-Mouth-Opening Displacement.

$$U(\text{Eq } 2)=\frac{1}{\left(\frac{EBv}{P}\right)^{\frac{1}{2}}+1}$$

$$U(\text{Eq } 3)=1-\left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}$$

$$U(\text{Eq } 4)=\frac{1}{\left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}+1}$$

Complete Coefficients in Table 2, 3, or 4

a/W	(EBv/P)	U(Eq 3)	a/W	Error	U(Eq 2)	a/W	Error	U(Eq 4)	a/W	error
0.2	6.98	-0.5050	0.2001	0.0001	0.2746	0.1999	-0.0001	0.3992	0.1998	-0.0002
0.3	12.36	-0.1305	0.3001	0.0001	0.2214	0.2999	-0.0001	0.4694	0.2998	-0.0002
0.4	21.04	0.1335	0.4002	0.0002	0.1790	0.3999	-0.0001	0.5358	0.3998	-0.0002
0.5	35.94	0.3369	0.5002	0.0002	0.1430	0.5000	0.0000	0.6013	0.4999	-0.0001
0.6	64.23	0.5040	0.5998	-0.0002	0.1109	0.5999	-0.0001	0.6685	0.5999	-0.0001
0.7	128.3	0.6491	0.7001	0.0001	0.0811	0.7000	0.0000	0.7402	0.6999	-0.0001

Complete Coefficients in Table 2, 3, or 4 to Three Significant Figures

a/W	(EBv/P)	U(Eq 3)	a/W	Error	U(Eq 2)	a/W	Error	U(Eq 4)	a/W	error
0.2	6.98	-0.5050	0.2003	0.0003	0.2746	0.1993	-0.0007	0.3992	0.2011	0.0011
0.3	12.36	-0.1305	0.3003	0.0003	0.2214	0.2994	-0.0006	0.4694	0.3011	0.0011
0.4	21.04	0.1335	0.4004	0.0004	0.1790	0.3994	-0.0006	0.5358	0.4009	0.0009
0.5	35.94	0.3369	0.5004	0.0004	0.1430	0.4996	-0.0004	0.6013	0.5003	0.0003
0.6	64.23	0.5040	0.6000	0.0000	0.1109	0.5996	-0.0004	0.6685	0.5993	-0.0007
0.7	128.3	0.6491	0.7003	0.0003	0.0811	0.6997	-0.0003	0.7402	0.6977	-0.0023

Table 6. Effects of Rounding the Calculated U Parameter on Crack Length Calculation for SE(B) Crack-Mouth-Opening Displacements. (Columns Headed 'X' Plcs are the U Parameters Rounded to 'X' Decimal Places)

$$U = \frac{1}{\left(\frac{EBv}{P}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	Six Plcs	a/W	Error	4 Plcs	a/W	Error	3 Plcs	a/W	Error	2 Plcs	a/W	Error
0.2	6.98	0.274644	0.1998	-0.0002	0.2746	0.1999	-0.0001	0.275	0.1992	-0.0008	0.27	0.2077	0.0077
0.3	12.36	0.221428	0.2999	-0.0001	0.2214	0.2999	-0.0001	0.221	0.3008	0.0008	0.22	0.3029	0.0029
0.4	21.04	0.176978	0.3999	-0.0001	0.1790	0.3999	-0.0001	0.179	0.3999	-0.0001	0.18	0.3973	-0.0027
0.5	35.94	0.142964	0.5001	0.0001	0.1430	0.5000	0.0000	0.14	0.5000	0.0000	0.14	0.5090	0.0090
0.6	64.23	0.110934	0.5998	-0.0002	0.1109	0.5999	-0.0001	0.111	0.5996	-0.0004	0.11	0.6028	0.0028
0.7	128.3	0.081125	0.6999	-0.0001	0.0811	0.7000	0.0000	0.081	0.7003	0.0003	0.08	0.7038	0.0038

$$U = \frac{1}{\left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	Six Plcs	a/W	Error	4 Plcs	a/W	Error	3 Plcs	a/W	Error	2 Plcs	a/W	Error
0.2	6.98	0.399196	0.1998	-0.0002	0.3992	0.1998	-0.0002	0.399	0.1995	-0.0005	0.40	0.2009	0.0009
0.3	12.36	0.469378	0.2998	-0.0002	0.4694	0.2998	-0.0002	0.469	0.2992	-0.0008	0.47	0.3007	0.0007
0.4	21.04	0.535760	0.3998	-0.0002	0.5358	0.3998	-0.0002	0.536	0.4001	0.0001	0.54	0.4062	0.0062
0.5	35.94	0.601299	0.4999	-0.0001	0.6013	0.4999	-0.0001	0.601	0.4994	-0.0006	0.60	0.4979	-0.0021
0.6	64.23	0.668461	0.5998	-0.0002	0.6685	0.5999	-0.0001	0.668	0.5992	-0.0008	0.67	0.6021	0.0021
0.7	128.3	0.740227	0.7000	0.0000	0.7402	0.6999	-0.0001	0.740	0.6997	-0.0003	0.74	0.6997	-0.0003

$$U = 1 - \left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}$$

a/W	(EBv/P)	Six Plcs	a/W	Error	4 Plcs	a/W	Error	3 Plcs	a/W	error	2 Plcs	a/W	Error
0.2	6.98	-0.505037	0.2001	0.0001	-0.5050	0.2001	0.0001	-0.505	0.2001	0.0001	-0.51	0.1990	-0.0010
0.3	12.36	-0.130481	0.3001	0.0001	-0.1305	0.3001	0.0001	-0.130	0.3003	0.0003	-0.13	0.3003	0.0003
0.4	21.04	0.133492	0.4002	0.0002	0.1335	0.4002	0.0002	0.133	0.4000	0.0000	0.13	0.3987	-0.0013
0.5	35.94	0.336934	0.5003	0.0003	0.3369	0.5002	0.0002	0.337	0.5003	0.0003	0.34	0.5019	0.0019
0.6	64.23	0.504027	0.5998	-0.0002	0.5040	0.5998	-0.0002	0.504	0.5998	-0.0002	0.50	0.5972	-0.0028
0.7	128.3	0.649063	0.7001	0.0001	0.6491	0.7001	0.0001	0.649	0.7001	0.0001	0.65	0.7008	0.0008

Table 7. Effects of a Relative Error of the Calculated U Parameter on Crack Length Calculation for SE(B) Crack-Mouth-Opening Displacements. (Columns Headed 'X' % are the Correct U Parameter Plus 'X' Percent)

$$U = \frac{1}{\left(\frac{EBv}{P}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	U	a/W	Error	0.1%	a/W	Error	1.0%	a/W	Error	5.0%	a/W	Error
0.2	6.98	0.274644	0.1998	-0.0002	0.2749	0.1994	-0.0006	0.2774	0.1952	-0.0048	0.2884	0.1771	-0.0229
0.3	12.36	0.221428	0.2999	-0.0001	0.2216	0.2994	-0.0006	0.2236	0.2952	-0.0048	0.2325	0.2770	-0.0230
0.4	21.04	0.178978	0.3999	-0.0001	0.1792	0.3995	-0.0005	0.1808	0.3953	-0.0047	0.1879	0.3772	-0.0228
0.5	35.94	0.142964	0.5001	0.0001	0.1431	0.4997	-0.0003	0.1444	0.4959	-0.0041	0.1501	0.4792	-0.0208
0.6	64.23	0.110934	0.5998	-0.0002	0.1110	0.5994	-0.0006	0.1120	0.5962	-0.0038	0.1165	0.5819	-0.0181
0.7	128.3	0.081125	0.6999	-0.0001	0.0812	0.6996	-0.0004	0.0819	0.6971	-0.0029	0.0852	0.6859	-0.0141

$$U = \frac{1}{\left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	U	a/W	Error	0.1%	a/W	Error	1.0%	a/W	Error	5.0%	a/W	Error
0.2	6.98	0.399196	0.1998	-0.0002	0.3996	0.2003	0.0003	0.4032	0.2053	0.0053	0.4192	0.2274	0.0274
0.3	12.36	0.469378	0.2998	-0.0002	0.4698	0.3005	0.0005	0.4741	0.3067	0.0067	0.4928	0.3347	0.0347
0.4	21.04	0.535760	0.3998	-0.0002	0.5363	0.4006	0.0006	0.5411	0.4080	0.0080	0.5625	0.4407	0.0407
0.5	35.94	0.601299	0.4999	-0.0001	0.6019	0.5008	0.0008	0.6073	0.5090	0.0090	0.6314	0.5452	0.0452
0.6	64.23	0.668461	0.5998	-0.0002	0.6691	0.6008	0.0008	0.6751	0.6095	0.0095	0.7019	0.6475	0.0475
0.7	128.3	0.740227	0.7000	0.0000	0.7410	0.7010	0.0010	0.7476	0.7098	0.0098	0.7772	0.7480	0.0480

$$U = 1 - \left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}$$

a/W	(EBv/P)	U	a/W	Error	0.1%	a/W	Error	1.0%	a/W	Error	5.0%	a/W	Error
0.2	6.98	-0.505037	0.2001	0.0001	-0.5055	0.2000	0.0000	-0.5101	0.1990	-0.0010	-0.5303	0.1945	-0.0055
0.3	12.36	-0.130481	0.3001	0.0001	-0.1306	0.3001	0.0001	-0.1318	0.2997	-0.0003	-0.1370	0.2980	-0.0020
0.4	21.04	0.133492	0.4002	0.0002	0.1336	0.4002	0.0002	0.1348	0.4008	0.0008	0.1402	0.4031	0.0031
0.5	35.94	0.336934	0.5003	0.0003	0.3373	0.5004	0.0004	0.3403	0.5021	0.0021	0.3538	0.5096	0.0096
0.6	64.23	0.504027	0.5998	-0.0002	0.5045	0.6002	0.0002	0.5091	0.6031	0.0031	0.5292	0.6163	0.0163
0.7	128.3	0.649063	0.7001	0.0001	0.6497	0.7006	0.0006	0.6556	0.7049	0.0049	0.6815	0.7244	0.0244

Table 8. Effects of an Absolute Error of the Calculated U Parameter on Crack Length Calculation for SE(B) Crack-Mouth-Opening Displacements. (Columns Headed '0.XXX1' are the Correct U Parameter Plus '0.XXX1')

$$U = \frac{1}{\left(\frac{EBv}{P}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	U	a/W	Error	0.0001	a/W	Error	0.001	a/W	Error	0.01	a/W	Error
0.2	6.98	0.274644	0.1998	-0.0002	0.2747	0.1997	-0.0003	0.2756	0.1982	-0.0018	0.2846	0.1832	-0.0168
0.3	12.36	0.221428	0.2999	-0.0001	0.2215	0.2997	-0.0003	0.2224	0.2978	-0.0022	0.2314	0.2791	-0.0209
0.4	21.04	0.178978	0.3999	-0.0001	0.1791	0.3997	-0.0003	0.1800	0.3974	-0.0026	0.1890	0.3746	-0.0254
0.5	35.94	0.142964	0.5001	0.0001	0.1431	0.4998	-0.0002	0.1440	0.4972	-0.0028	0.1530	0.4710	-0.0290
0.6	64.23	0.110934	0.5998	-0.0002	0.1110	0.5995	-0.0005	0.1119	0.5965	-0.0035	0.1209	0.5677	-0.0323
0.7	128.3	0.081125	0.6999	-0.0001	0.0812	0.6996	-0.0004	0.0821	0.6964	-0.0036	0.0911	0.6656	-0.0344

$$U = \frac{1}{\left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}} + 1}$$

a/W	(EBv/P)	U	a/W	Error	0.0001	a/W	Error	0.001	a/W	Error	0.01	a/W	Error
0.2	6.98	0.399196	0.1998	-0.0002	0.3993	0.1999	-0.0001	0.4002	0.2012	0.0012	0.4092	0.2135	0.0135
0.3	12.36	0.469378	0.2998	-0.0002	0.4695	0.2999	-0.0001	0.4704	0.3013	0.0013	0.4794	0.3146	0.0146
0.4	21.04	0.535760	0.3998	-0.0002	0.5359	0.3999	-0.0001	0.5368	0.4013	0.0013	0.5458	0.4151	0.0151
0.5	35.94	0.601299	0.4999	-0.0001	0.6014	0.5000	0.0000	0.6023	0.5014	0.0014	0.6113	0.5150	0.0150
0.6	64.23	0.668461	0.5998	-0.0002	0.6686	0.6000	0.0000	0.6695	0.6013	0.0013	0.6785	0.6143	0.0143
0.7	128.3	0.740227	0.7000	0.0000	0.7403	0.7001	0.0001	0.7412	0.7013	0.0013	0.7502	0.7132	0.0132

$$U = 1 - \left(\frac{3.95(S/W)P}{EBv}\right)^{\frac{1}{2}}$$

a/W	(EBv/P)	U	a/W	Error	0.0001	a/W	Error	0.001	a/W	Error	0.01	a/W	Error
0.2	6.98	-0.505037	0.2001	0.0001	-0.5049	0.2001	0.0001	-0.5040	0.2003	0.0003	-0.4950	0.2024	0.0024
0.3	12.36	-0.130481	0.3001	0.0001	-0.1304	0.3002	0.0002	-0.1295	0.3005	0.0005	-0.1205	0.3034	0.0034
0.4	21.04	0.133492	0.4002	0.0002	0.1336	0.4002	0.0002	0.1345	0.4006	0.0006	0.1435	0.4046	0.0046
0.5	35.94	0.336934	0.5003	0.0003	0.3370	0.5003	0.0003	0.3379	0.5008	0.0008	0.3469	0.5058	0.0058
0.6	64.23	0.504027	0.5998	-0.0002	0.5041	0.5999	-0.0001	0.5050	0.6005	0.0005	0.5140	0.6063	0.0063
0.7	128.3	0.649063	0.7001	0.0001	0.6492	0.7002	0.0002	0.6501	0.7008	0.0008	0.6591	0.7075	0.0075

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